

TABLE 8 Critical Values of Smaller Rank Sum for the Wilcoxon Mann-Whitney Test

n_2	α for Two-Sided Test		α for One-Sided Test		n_1 (Smaller Sample)																				
	.10	.05	.10	.05	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
3	.20	.10	.10	.05			3	7																	
	.10	.05	.05	.025			6																		
	.05	.025	.01	.005																					
4	.20	.10	.10	.05		3	7	13																	
	.10	.05	.05	.025		6	6	10																	
	.05	.025	.01	.005																					
5	.20	.10	.10	.05		4	8	14	20																
	.10	.05	.05	.025		3	7	12	19																
	.05	.025	.01	.005			6	11	17	15															
6	.20	.10	.10	.05		4	9	15	22	30															
	.10	.05	.05	.025		3	8	13	20	28															
	.05	.025	.01	.005			7	12	18	26															
7	.20	.10	.10	.05		4	10	16	23	32	41														
	.10	.05	.05	.025		3	8	14	21	29	39														
	.05	.025	.01	.005			7	13	20	27	36														
8	.20	.10	.10	.05		5	11	17	25	34	44	55													
	.10	.05	.05	.025		4	9	15	23	31	41	51													
	.05	.025	.01	.005		3	8	14	21	29	38	49													
9	.20	.10	.10	.05		1	5	11	19	27	36	46	58	70											
	.10	.05	.05	.025		4	*10	16	24	33	43	54	66												
	.05	.025	.01	.005		3	8	14	22	31	40	51	62												
	.01					6	6	11	18	26	35	45	56												

(continued)

TABLE 8 Critical Values of Smaller Rank Sum for the Wilcoxon Mann-Whitney Test (Continued)

n_2	α for Two-Sided Test		α for One-Sided Test		n_1 (Smaller Sample)																				
	.10	.05	.10	.05	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10	.20	.10	.10	.05	1	6	12	20	28	38	49	60	73	87											
	.10	.05	.05	.025		4	10	17	26	35	45	56	69	82											
	.05	.025	.01	.005		3	9	15	23	32	42	53	65	78											
11	.20	.10	.10	.05	1	6	13	21	30	40	51	63	76	91	106										
	.10	.05	.05	.025		4	11	18	27	37	47	59	72	86	100										
	.05	.025	.01	.005		3	9	16	24	34	44	55	68	81	96										
12	.20	.10	.10	.05	1	7	14	22	32	42	54	66	80	94	110	127									
	.10	.05	.05	.025		5	11	19	28	38	49	62	75	89	104	120									
	.05	.025	.01	.005		4	10	17	26	35	46	58	71	84	99	115									
13	.20	.10	.10	.05	1	7	15	23	33	44	56	69	83	98	114	131	149								
	.10	.05	.05	.025		5	12	20	30	40	52	64	78	92	108	125	142								
	.05	.025	.01	.005		4	10	18	27	37	48	60	73	88	103	119	136								
14	.20	.10	.10	.05	1	*8	16	25	35	46	59	72	86	102	118	136	154	174							
	.10	.05	.05	.025		*6	13	21	31	42	54	67	81	96	112	129	147	166							
	.05	.025	.01	.005		4	11	19	28	38	50	62	76	91	106	123	141	160							
15	.20	.10	.10	.05	1	8	16	26	37	48	61	75	90	106	123	141	159	179	200						
	.10	.05	.05	.025		6	13	22	33	44	56	69	84	99	116	133	152	171	192						
	.05	.025	.01	.005		4	11	20	29	40	52	65	79	94	110	127	145	164	184						
16	.20	.10	.10	.05	1	8	17	27	38	50	64	78	93	109	127	145	165	185	206	229					
	.10	.05	.05	.025		6	14	24	34	46	58	72	87	103	120	138	156	176	197	219					
	.05	.025	.01	.005		4	12	21	30	42	54	67	82	97	113	131	150	169	190	211					
	.01					8	15	24	34	46	58	72	86	102	119	136	155	175	196						

TABLE 8 Critical Values of Smaller Rank Sum for the Wilcoxon Mann-Whitney Test (Continued)

n_2	n_1 (Smaller Sample)																						
	α for Two-Sided Test	α for One-Sided Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
17	.20	.10	1	9	18	28	40	52	66	81	97	113	131	150	170	190	212	235	259				
	.10	.05		6	15	25	35	47	61	75	90	106	123	142	161	182	203	225	249				
	.05	.025		5	12	21	32	43	56	70	84	100	117	135	154	174	195	217	240				
	.01	.005			8	16	25	36	47	60	74	89	105	122	140	159	180	201	223				
18	.20	.10	1	9	19	30	42	55	69	84	100	117	135	155	175	196	218	242	266	291			
	.10	.05		7	15	26	37	49	63	77	93	110	127	146	166	187	208	231	255	280			
	.05	.025		5	13	22	33	45	58	72	87	103	121	139	158	179	200	222	246	270			
	.01	.005			8	16	26	37	49	62	76	92	108	125	144	163	184	206	228	252			
19	.20	.10	2	10	20	31	43	57	71	87	103	121	139	159	180	202	224	248	273	299	325		
	.10	.05	1	7	16	27	38	51	65	80	96	113	131	150	171	192	214	237	262	287	313		
	.05	.025		5	13	23	34	46	60	74	90	107	124	143	163	183	205	228	252	277	303		
	.01	.005			9	17	27	38	50	64	78	94	111	129	148	168	189	210	234	258	283		
20	.20	.10	2	10	21	32	45	59	74	90	107	125	144	164	185	207	230	255	280	306	333	361	
	.10	.05	1	7	17	28	40	53	67	83	99	117	135	155	175	197	220	243	268	294	320	348	
	.05	.025		5	14	24	35	48	62	77	93	110	128	147	167	188	210	234	258	283	309	337	
	.01	.005			9	18	28	39	52	66	81	97	114	132	151	172	193	215	239	263	289	315	

For larger values of n_1 and n_2 , critical values are given to a good approximation by the formula:

$$\frac{n_1(n_1 + n_2 + 1) - z \left\{ \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} \right\}^{1/2}}{2}$$

where $z = 1.28$ for $\alpha = .20$ (two-sided test)

$z = 1.64$ for $\alpha = .10$ (two-sided test)

$z = 1.96$ for $\alpha = .05$ (two-sided test)

$z = 2.58$ for $\alpha = .01$ (two-sided test)

* Values have been corrected to the values given by D. B. Owen, *Handbook of Statistical Tables*, copyright 1962, Addison-Wesley Publishing Co., Inc.

TABLE 9 Critical Values of $W_c(n)$ for the Wilcoxon Signed-Ranks Test

W_c is the integer such that the probability that $W \leq W_c$ is closest to α . For example, for $n = 8$, $P(W \leq 3) = .020$ and $P(W \leq 4) = .027$; therefore, $W_{.027}(8) = 4$.

n	α for One-Sided Test		
	.025	.01	.005
	α for Two-Sided Test		
	.05	.02	.01
6	0	—	—
7	2	0	—
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	20
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	38
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	89	77	68

For large n ,

$$W_c(n) \approx \frac{n(n+1)}{4} - z_{1-\alpha} \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

approximately, where z is given in Table 2.